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Exam.Code:1014
Sub. Code: 7751

1078
M.E. (Mechanical Engineering)
First Semester
MME-103: Continuum Mechanics

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, selecting atleast two questions from each Part. Use usual notations and symbols for derivations. Assume suitably missing data if any.*

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Part A

Q1. (a) Which of the following equations are valid or invalid using the rules of index expression. Provide explanation for the invalid ones.

i. $S_{ij} = s_{ij} - s_{kk}\delta_{ij}$

ii. $\rho\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_j}C_{ijkl}\frac{\partial u_k}{\partial x_l}$

Iii. $\epsilon_{ijk}\epsilon_{kkj} = 0$

(b) Given $R = \sqrt{x_k x_k}$ calculate $\frac{\partial \log R}{\partial x_i}$ and $\frac{\partial^2 \log R}{\partial x_i \partial x_i}$

Q2. (a) Given S is symmetric and W is skew. Determine S·W.

(b) Let S be a non-singular second order tensor with invariants I_1, I_2, I_3. Show that $S^{-1} = (S^2 - I_1 S + I_2 I)/I_3$.

Q3. Show that the Lagrange strain E, the right Cauchy-Green deformation tensor C and the right stretch tensor U have the same principal directions (eigenvectors). Similarly, show that e, B, and V have the same principal directions.

Q4. Parabolic coordinates specify the position of a point using three parametric coordinates (u, v, θ) as

$$r = \text{uv}(\cos\theta\, i + \sin\theta\, j) + \frac{1}{2}(u^2 + v^2)k.$$

Find the components of normalized basis vectors for this coordinate system, show that they are orthogonal, and calculate their derivatives with respect to (u, v, θ), express your answers in (e_u, e_v, e_θ) coordinates.

Part B

Q5. A rubber band has an initial length of L. One end of the band is held fixed. For time $t>0$ the other end is pulled as constant speed v_0. Let X denote position in the undeformed configuration and x denote position in the deformed configuration. Assume one dimensional deformation. Write the position x of a material particle as a function of initial position X and time t. Hence determine the velocity distribution as both a function of X and a function of x. Also find the deformation gradient and velocity gradient. Next suppose a fly walks along the rubber band with a speed w relative to the band. Calculate the acceleration of the fly. Suppose the fly is at $X=x=0$ at time $t=0$, find how long it will take for the fly to walk to the other end of the rubber band.

P.T.O.

Q6. For the Cauchy stress tensor with components

$$\begin{bmatrix} 100 & 250 & 0 \\ 250 & 200 & 0 \\ 0 & 0 & 300 \end{bmatrix} \text{MPa}$$

Compute the traction vector acting on the plane with normal

$n = (e1-e2)/\sqrt{2}$. Also compute the principal stresses, hydrostatic stress, deviatoric stress tensor, and the von-Mises equivalent stress.

Q7. Starting with the statement of the second law of thermodynamics for a control volume, derive the local form.

Q8. What is Superposed Rigid Body Motion and how are various quantities invariant under it.

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