Exam. Code: 0905
Sub. Code: 7884

1078
B. Engg.-1st Semester
AS-101: Engg. Mathematics-I
(Common)

Time allowed: 3 Hours | Max. Marks: 50

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

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UNIT-I

I. (a) At what points in the plane or space are the functions continuous:

(i) $f(x,y) = \sin\dfrac{1}{xy}$,

(ii) $f(x,y) = \dfrac{x+y}{2+\cos x}$

(iii) $f(x,y,z) = \dfrac{1}{x^2+z^2-1}$

(b) If $f(u,v,w)$ is differentiable and $u = x-y$, $v = y-z$ and $w = z-x$, prove that $\dfrac{\partial f}{\partial x}+\dfrac{\partial f}{\partial y}+\dfrac{\partial f}{\partial z}=0$ (5+5)

II. (a) Let $T = g(x,y)$ be the temperature at the point (x,y) on the ellipse $x = 2\sqrt{2}\cos t$, $y = \sqrt{2}\sin t$, $0 \le t \le 2\pi$ and suppose that $\dfrac{\partial T}{\partial x} = y$, $\dfrac{\partial T}{\partial y} = x$.

(i) Locate the maximum and minimum temperatures on the ellipse by examining $\dfrac{dT}{dt}$ *and* $\dfrac{d^2T}{dt^2}$.

(ii) Suppose that $T = xy-2$. Find the maximum and minimum value of T on the ellipse.

(b) If $u = f(x,y)$ *and* $x = r\cos\theta, y = r\sin\theta$, then prove that

$$\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2=\left(\frac{\partial f}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$$ (5+5)

III. (a) Expand $f(x,y) = \tan^{-1}(xy)$ in power of (x-1) and (y-1) upto second degree terms. Hence, compute f(1.1, 0.8).

(b) Find the area of the region in the first quadrant that in bounded above by $y = \sqrt{x}$ and below by x-axis and the line y=x-2. (5+5)

P.T.O.

IV. (a) Change the order of integration and evaluate:

$$I = \int_{0}^{p}\int_{x^2}^{2-x} xydydx$$

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (5+

UNIT-II

V. (a) Find the curvature for the helix $\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k}$, $a,b \geq 0, \quad a^2+b^2 \neq 0$.

(b) Find the disectional derivative of $\varphi(x,y,z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector; $\hat{i}+2\hat{j}+2\hat{k}$. (5+5

VI. (a) Find $div\vec{F}$ and $curl\vec{F}$ when $\vec{F} = grad(x^3+y^3+z^3-3xyz)$.

(b) Find the flux of $\vec{F} = (x-y)\hat{i} + x\hat{j}$ across the circle: $x^2+y^2=1$, in the xy plane. (5+5

VII. (a) State Green's theorem. Apply it to evaluate the integr $\oint_C (6y+x)dx + (y+2x)dy$, where C: the circle $(x-2)^2+(y-3)^2=4$.

(b) Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$, *if* $\vec{F} = xz\hat{i} + xy\hat{j} + 3xz\hat{k}$ and in the boundary of the portion of the plane 2x+y+z=2 in the first octan traversed counter clockwise as viewed from above. (5+5

VIII. (a) Prove that $\vec{F} = (e^x\cos y + yz)\hat{i} + (xz - e^x\sin y)\hat{j} + (xy+z)\hat{k}$ conservative and find a potential for it.

(b) State Gauss divergence theorem. Evaluate both sides of it for the fiel $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2+y^2+z^2=4$. (5+5

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