3rd 35

Exam. Code: 0939
Sub. Code: 7045

1128

Bachelor of Engineering (Mechanical Engineering)
3rd Semester
AS – 301: Maths - 3

Time allowed: 3 Hours **Max. Marks: 50**

Note: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Unit.

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I. Attempt the following questions:-

a) Define limit of a sequence, convergent and bounded sequence with suitable examples. Give an example of a sequence which is not convergent.

b) Define an alternating series and state the test used for checking its convergence. When an alternating series is said to be absolutely convergent and conditionally convergent? Give example.

c) Explain partial and complete pivoting in connection with Gauss elimination method.

d) Prove that $f(z) = \bar{Z}$ is not analytic at any point.

e) Locate and classify the singular points of $f(z) = \frac{Z - Sin\ Z}{Z^3}$. (5×2)

UNIT – I

II. a) Which of the following sequences $\{a_n\}$ converge and which diverge? Find the limit of each convergent sequence:-

i) $a_n = n - \sqrt{n^2 - n}$

ii) $a_n = \left(\frac{5n+1}{5n-1}\right)^n$

iii) $a_n = Sinh\ (\ell nn)$

iv) $a_n = \ell nn - \ell n(n+1)$

b) Discuss the convergence or divergence of the following series:-

i) $\sum_{n=1}^{\infty} \frac{Cos n\pi}{n}$ ii) $\sum_{n=1}^{\infty} \frac{\tanh n}{n^2}$

iii) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$ iv) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{3/2}}$ (10)

III. a) State and prove Cauchy's integral test. Hence, discuss the convergence or divergence of the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$,(p a real constant).

b) Prove that the vectors $\vec{V_1} = (2,1,1), \vec{V_2} = (1,2,2), \vec{V_3} = (1,1,1)$, are linearly independent.

c) Solve the linear system by Gauss elimination method:-
$2x + y + z = 10, 3x + 2y + 2z = 18$, and $x + 4y + 9z = 16$ (10)

P.T.O.

Sub. Code: 7045

IV. a) Find the matrix P that diagonalizes $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$. Also determine $P^{-1}AP$.

b) Using Cayley-Hamilton theorem, find A^{-1} where $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.

c) Examine whether $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is similar to $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. (10)

UNIT – II

V. a) If $f(z)$ is analytic in a domain D and if $(z)|$ is a non-zero constant in D, then prove that $f(z)$ is constant in D.

b) Prove that the function $u(x,y) = 2x + y^3 - 3x^2y$ is harmonic. Find conjugate harmonic function $v(x,y)$ and the corresponding analytic function $f(z)$. (10)

VI. a) Prove that $W = Sin\ Z$ is not a bounded function.

b) Discuss the mapping $W = e^z$.

c) Find two bilinear transformations whose fixed points are 1 and 2. (10)

VII. a) Exapand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for:-

i) $|< |z| < 3$

ii) $0 < |z+1| < 2$

iii) $|z| > 3$

b) Using the complex variable technique, evaluate $I = \int_0^{2\pi} \frac{d\theta}{2+cos\theta}$ (10)

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