3rd 8

Exam. Code: 0939
Sub. Code: 6699

1128

Bachelor of Engineering (Mechanical Engineering)
3rd Semester
AS – 301: Engineering Mathematics – III

Time allowed: 3 Hours **Max. Marks: 50**

Note: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Unit.

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I. Attempt the following questions:-

a) Prove that for every convergent series $\sum_{n=1}^{\infty} u_n$, $\lim_{n\to\infty} u_n = 0$, but the converse is not true.

b) Define Eigen value problem of matrices. Find the eigen values of A^T, A^{-1}, where $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 11 \end{bmatrix}$

c) Prove that similar matrices have the same Eigen values, but the converse is not true.

d) Does the $\lim_{z\to 0} \frac{z}{\bar{z}}$ exist? If yes, find it. If not, explain it.

e) Define residue and find the same for $f(z) = z.\cos\frac{1}{z}$ at $z = 0$. (5×2)

UNIT – I

II. a) Which of the following sequences $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence:-

i) $a_n = \int_1^n \frac{1}{x^p} dx, p > 1$

ii) $a_n = \frac{(\frac{10}{11})^n}{(\frac{9}{10})^n + (\frac{11}{12})^n}$

iii) $a_n = n - \sqrt{n^2 - n}$

iv) $a_n = \sinh(\ln n)$

b) State and prove Cauchy's integral test. Hence, discuss the convergence or divergence of the p-series:- $\sum_{n=1}^{\infty} \frac{1}{n^p}$. (5,5)

III. a) Find the radius and interval of convergence for the series:- $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$. For what values of x does the series converge; **i)** absolutely, **ii)** Conditionally?

b) Express the matrix $A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ as a linear combination of the matrices:-

$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

c) Find the rank of the matrix:-

$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 3 & -1 & 1 \\ 5 & 4 & 2 & 0 \end{bmatrix}$ (4,3,3)

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IV. a) Solve the following system by Gauss elimination method:-
$2x + y + z = 10, \quad 3x + 2y + 2z = 18, \quad x + 4y + 9z = 16$

b) Examine whether $A = \begin{bmatrix} 7 & 7 \\ -2 & 0 \end{bmatrix}$, is similar to $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ or not?

c) The eigen vectors of a 3×3 matrix corresponding to the eigen values 2, 2, 4 are $(-2, 1, 0)^T$, $(-1, 0, 1)^T$, $(1, 0, 1)^T$, respectively. Find the matrix A. (4,3,3)

UNIT – II

V. a) Prove that $W = Cos\ Z$ is not a bounded function.

b) Prove that the function $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at $z = 0$, even though C-R equations are satisfied there.

c) Prove that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic everywhere. Also find the conjugate harmonic of $u(x, y)$. (3,4,3)

VI. a) Find the Laurent's series expansion of $f(z) = \frac{1}{z - z^3}$ in the region $1 < |z + 1| < 2$.

b) Explain and write different type of isolated singularities. Give one example of each. Also prove that if an analytic function $w = f(z)$ has a pole of order m at $z = z_0$, then $\frac{1}{f(z)}$ has a zero of order m at $z = z_0$. (5,5)

VII. a) Discuss the mapping $w = z + \frac{1}{z}$

b) Using residue theorem, prove that:-

$$\int_0^{2\pi} \frac{d\theta}{1 - 2p\cos\theta + p^2} = \frac{2\pi}{1 - p^2}, \quad 0 < |p| < 1$$ (5,5)

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