Exam. Code: 0921
Sub. Code: 6831

1128

B. Engg. (Information Technology)

3rd Semester

MATHS-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours | Max. Marks: 50

NOTE: *Attempt **five** questions in all, including Q. No. I (Unit-I) which is compulsory and selecting atleast two questions each from Unit II-III.*

-*-*-*-

UNIT-I

I. (a) Find the conditions on a,b,c so that $\vec{v} = (a,b,c)$ *in* IR^3 belongs to linear span of vectors $(1,2,0)$ $(-1,1,2)$ $(3,0,-4)$.

(b) Let $F: IR^2 \to IR^2$ be defined by $F(x,y) = (2x+3y, 4x-5y)$. Find the matrix representation of F relative to the basis $\{(1,-2),(2,-5)\}$ *of* IR^2.

(c) A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the probability that actually there was six?

(d) A box contains 'a' white and 'b' black balls. 'c' balls are drawn at random. Find the expected value of the number of white balls drawn. (3+2+3+2)

UNIT-II

II. (a) Find the coordinate vector of $3t^3 - 4t^2 + 2t - 5$ relative to the basis $\{(t-1)^3, (t-1)^2, (t-1), 1\}$ *of* $P_3(t)$.

(b) Find a linear mapping $F: IR^2 \to IR^2$ whose image is spanned by $(1,2,3)$ *and* $(4,5,6)$. (5+5)

III. (a) Find the characteristics polynomial of matrix

$$\begin{bmatrix} 2 & 5 & 1 & 1 \\ 1 & 4 & 2 & 2 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & 2 & 3 \end{bmatrix}$$

(b) Reduce the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{bmatrix}$ to echelon form. (5+5)

IV. (a) Verify the characteristic equation for the matrix $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.

(b) State Rank-Nullity theorem and verify it form the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (2x + y - 2z,\ 2x + 3y - 4z\ \ x + y - z)$. (5+5)

UNIT-III

V. (a) Two random variables X and Y have the following probability density function $f(x, y) = \begin{cases} 2 - x - y & ; \ 0 \leq x \leq 1 \quad 0 \leq y \leq 1 \\ 0 & ; \ otherwise \end{cases}$,

Find Var(X) and Var(Y). Also find covariance between X and Y.

(b) After correcting 50 pages of the proof of a book, the proof reader finds that there are, on the average, 2 errors per 5 pages. How many pages would one expect to find with 0,1,2,3 and 4 errors, in 1000 pages of the first print of the book? (5+5)

VI. (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

(b) For the joint probability distribution of two random variables X and Y given below:

	1	2	3	4	Total
1	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{9}{36}$
3	$\frac{5}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{8}{36}$
4	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{9}{36}$
Total	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	1

Find conditional distribution of X given the value of Y=1 and that of Y given the value of X=2. (5+5)

VII. (a) A symmetric dic is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

(b) What is the probability that atleast two out of n people have the same birthday? Assume 365 days in a year and all days are equally likely. (5+5)

-*-*-*-