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Exam. Code: 0933
Sub. Code: 6974

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Bachelor of Engineering (Electrical and Electronics Engineering)
3^{rd} Semester
MATHS – 301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

Note: Attempt any five questions, including Question No. 1 which is compulsory and selecting at least two questions from each Unit.

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I. Attempt the following questions:-

a) Define a vector space with suitable example.

b) Show that the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ is a linear combination of:- $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $y = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $Z = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

c) Is $W = \{(x, y)1x = 3y + 1\}$ is a sub-space of R^2 ?

d) Is $S = \{(0,0,0\}$ Lim early dependent? Justify.

e) Prove that similar matrices gave same eigen values.

f) Find the values of e^z for which z is a pure imaginary. List out the differences and similarities between e^x and e^z.

g) Prove that $f(z) = |z|^2$ is not analytic at any point.

h) If $f(z)$ has a simple pole at $z = a$, then Res. $f(a) = \lim_{z\to a} (z-a) f(z)$.

i) Find the image of $|z + 1|$ under the mapping $w = \frac{1}{z}$

j) Find the fixed and critical points of the mapping $w = z^3$ (10×1)

UNIT – I

II. a) Solve the following system by Gauss elimination method:-
$2x + y + z = 10,\ 3x + 2y + 2z = 18,\ x + 4y + 9z = 16$

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$

c) Prove that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by: $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space. (3,3,4)

III. a) Determine which of the following are subspaces of R^3:-

i) all the vectors of the form (a, b, c), where b = a + c.

ii) all the vectors of the form (a, b, c), where b = a + c + 1.

b) State rank-nullity theorem. Verify it for the linear transformation $T: R^3 \longrightarrow R^2$ defined by $T(x, y, z) = (x + y + z, x + y)$

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c) Find the basis for the column space of:-

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

(3,4,3)

IV. a) Using Cayley – Hamilton theorem, find A^{-1} where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

b) Let $T: R^3 \longrightarrow R^3$ be a linear operator defined by:- $T(x, y, z) = (3x, x - y, 2x + y + z)$. Show that T is invertible and find T^{-1}.

c) Consider the linear transformation T on R^2 defined by $T(x, y) = (2x - 3y, x + 4y)$ and the following bases of R^2:- $S = \{(1,0), (0,1)\}$ and $S^1\{(1,3), (2,5)\}$.

i) Find the matrix A representing T relative to the bases S and S^1.

ii) Find the change of basis matrix from S to S^1. (3,3,4)

UNIT – II

V. a) Find all values of Z which satisfies $e^{\frac{1}{z}} = 1 - i$.

b) If *f(z)* is analytic is a domain D and $|f(z)|$ is a non-zero constant in D, then prove that *f(z)* is constant in D.

c) If *u(x,y)* is a harmonic function, then prove that $w = u^2$ is not harmonic unless u is a constant. (3,4,3)

VI. a) Obtain Taylor's / Laurent's series expansion of $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$, which are valid in :-

i) $|z| < 1$

ii) $1 < |z| < 4$

iii) $|z| > 4$

b) Find the sum of the residues of the function $f(z) = \frac{Sin\ Z}{Z\ Cos\ Z}$ at its pole inside the circle $|z| = 2$ (5,5)

VII. a) Evaluate:- $\int_0^{\pi} \frac{3d\theta}{9 + Sin^2\theta}$ using complex integration.

b) Examine the exponential transformation $w = e^z$ (5,5)

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