

1128
B. E. (Computer Science and Engineering)
Third Semester
CS-303: Discrete Structures

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

1. Answer the following:

- Suppose R is a relation on a finite set A having n elements. What will be the number of relations on A?
- In how many ways a group of 5 students can be selected from 6 boys and 5 girls, consisting of 3 boys and 2 girls?
- Express the statement using quantifiers. "Every student in your school has a computer or has a friend who has a computer."
- Show that $(p \wedge (\sim p \vee q)) \wedge \sim q$ is tautology or contradiction.
- If $A = \{2, 3, 4\}$ and $B = \{5, 6\}$. Determine all functions from A to B.
- If $f(x) = x^2$, $-3 \leq x \leq 3$, find its range.
- Define order and size of a graph.
- Give an example of graph which is Hamiltonian but not Eulerian and vice versa.
- Define Monoids with example.
- Explain Homomorphism with example.

(10x1=10)

Section A:

- What is Partially Ordered Set? Let $S = \{a, b, c\}$ and $A = P(S)$. Draw the Hasse diagram of the poset A with the partial order \subseteq (set inclusion).
 - Let $A = B = \{1, 2, 3, 4\}$. Define function $f: A \rightarrow B$ (if possible) such that
 - f is one-to-one and onto.
 - f is neither one-to-one nor onto
 - f is onto but not one-to-one.
 - f is one-to-one but not onto.

(04+06)

- What is the minimum number of students required in a class to be sure that at least 6 will receive the same grade if there are five possible grades A, B, C, D and F?
 - Consider f, g and h, all the functions on the integers by $f(n) = n^2$, $g(n) = n + 1$ and $h(n) = n - 1$. Determine i) hofog ii) gofoh iii) fogoh iv) hofof.
 - Consider $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x - 7$. Show that f is both injective and surjective.

(02+04+04)

4. a) Test the validity of argument:

"If it rains tomorrow, I will carry my umbrella, if its cloth is mended. It will rain tomorrow and the cloth will not be mended. Therefore I will not carry my umbrella"

- Prove the validity of following arguments without using truth table.
 $(p \wedge q) \rightarrow r, (r \rightarrow q), (r \rightarrow q) \rightarrow (q \wedge r) \vdash (p \wedge q) \rightarrow (q \wedge r)$
- Write the following arguments using quantifiers, variables and predicate symbols.
 - All birds can fly.
 - Some men are genius.
 - Not all birds can fly.
 - There is a student who likes mathematics but not geography.

(04+02+04)

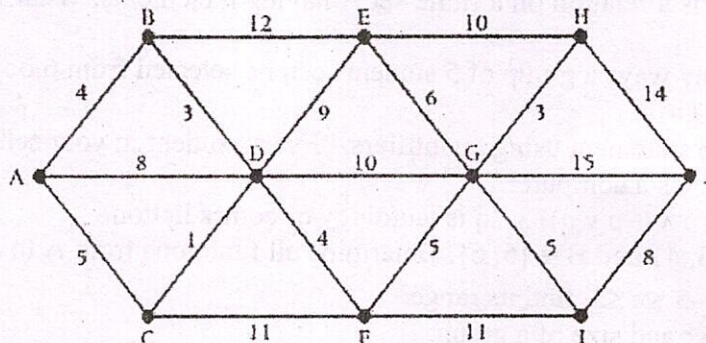
Section B:

5. a) Define order and degree of a recurrence relation. Solve the following recurrence relation by substitution method:

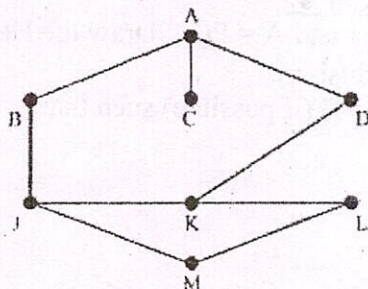
$$t_n = \begin{cases} 2t_{n-1} + 1, & n \geq 2 \\ 1, & n = 1 \end{cases}$$

- b) Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$ by the generating function method with initial conditions, $a_0 = 2$ and $a_1 = 3$.
 c) How many committees of five with a given chairperson can be selected from 12 persons? (04+04+02)

6. a) Determine the shortest path between the vertices A to J of the graph given in following figure:



- b) Consider the graph G in following figure.



- i) Find the adjacency structure of G.
 ii) Find the order of the vertices of G is processed using a DFS algorithm beginning at vertex A.

(06+04)

7. a) Consider a ring $(R, +, *)$ defined by $a * a = a$. Determine whether the ring is commutative or not.
 b) Let A be the set of 2×2 matrices. Show that semigroups, $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}, \times$ and $(R, +)$ are isomorphism

(05+05)