3rd. C.

Exam. Code: 0907
Sub. Code: 6695

1128

Bachelor of Engineering (Bio-Technology)
3rd Semester
MATHS – 302: Linear Algebra and Operations Research
(Common with IT)

Time allowed: 3 Hours **Max. Marks: 50**

Note: Attempt any <u>five</u> questions, including Question No. 1 which is compulsory and selecting at least two questions from each Unit. Use of Calculator is allowed.

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I. Attempt the following questions:-

a) Define a vector space with suitable example. Give an example of finite and infinite dimensional vector space.

b) State Cayley – Hamilton theorem. Write any two application of it.

c) Define a diagonalizable matrix. Why we diagonalize a given matrix? Write a matrix that is not diagonalizable with justification.

d) While solving a LPP: Maximize $Z = CX$ such that $AX = b$, $x \geq 0$; what indicates the following:-
   i) Presence of redundant constraints.
   ii) No feasible solution.

e) Give any three applications of duality theory. (5×2)

UNIT – I

II. a) Express the matrix:- $\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ as a linear combination of the matrices:-

$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$.

b) Find the rank of the matrix:- $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$

c) Determine whether the set of vectors will be a basis of R^3.

$\vec{V}_1 = (1,2,1)$, $\vec{V}_2 = (2,9,0)$, $\vec{V}_3 = (3,3,4)$. (3,3,4)

III. a) Verify Cayley – Hamilton theorem for the matrix:- $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1} using it.

b) Find the matrix P that diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$. (5,5)

IV. a) Consider the system of equations:-
$x_1 + 2x_2 + 4x_3 + x_4 = 7$, $2x_1 - x_2 + 3x_3 - 2x_4 = 4$. Here $x = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$ is a feasible solution. Reduce the feasible solution to two different basic feasible solutions.

b) Use two phase simplex method to solve:-
Maximize: $Z = 5x_1 - 4x_2 + 3x_3$
Subject to the constraints:- (5,5)
$2x_1 + x_2 - 6x_3 = 20$, $6x_1 + 5x_2 + 10x_3 \leq 76$, $8x_1 - 3x_2 + 6x_3 \leq 50$, $x_1, x_2, x_3 \geq 0$

UNIT – II

V. a) Solve the following LPP using the result of its dual problem:-
Minimize:- $Z = 24x_1 + 30x_2$,
Subject to the constraints:-
$2x_1 + 3x_2 \geq 10$, $4x_1 + 9x_2 \geq 15$, $6x_1 + 6x_2 \geq 20$, $x_1, x_2 \geq 0$

b) Use dual simplex method to solve:-
Minimize:- $Z = 3x_1 + x_2$
Subject to the constraints:- $x_1 + x_2 \geq 1$, $2x_1 + 3x_2 \geq 2$, $x_1, x_2 \geq 0$ (5,5)

P.T.O.

VI. a) Find the initial basic feasible solution to the following transportation problem:-

i) North-West corner cell method.

ii) Least cost cell method.

		TO			
		1	**2**	**3**	**SUPPLY**
FROM	**1**	2	7	4	5
	2	3	3	1	8
	3	5	4	7	7
	4	1	6	2	14
DEMAND		2	9	18	

State which of the method is better?

b) Solve the following assignment problem:-

	1	**2**	**3**	**4**
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

(5,5)

VII. a) Compare CPM and PERT explaining similarities and mentioning where they mainly differ.

b) A small project consists of seven activities for which the relevant data are given below:-

Activity	**Preceding Activities**	**Activity Duration (weeks)**
A	-	4
B	-	7
C	-	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

i) Draw the network and find the project completion time.

ii) Calculate total float for each of the activities and highlight the critical path.

iii) Draw the time scaled diagram. (5,5)

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