23.

Exam.Code:1014
Sub. Code: 7751

1128
M. E. (Mechanical Engineering)
First Semester
MME-103: Continuum Mechanics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Part. Use usual notations and symbols for derivations. Assume suitable missing data, if any.

x-x-x

Section A

Q.1 (a) In direct notation prove that $\mathbf{S}(\mathbf{a} \otimes \mathbf{b}) = (\mathbf{Sa}) \otimes \mathbf{b}$. (b) Prove that the Cartesian componensts of the product of two tensors $\mathbf{S}$ and $\mathbf{T}$ are $(\mathbf{ST})_{ij} = S_{ik}T_{kj}$.

Q.2 Prove in direct notation that $\frac{d}{d\mathbf{A}} I_2(\mathbf{A}) = I_1(\mathbf{A})\mathbf{I} - \mathbf{A}^{\mathrm{T}}$.

Q.3 Given the motion $x_1 = 2X_2$, $x_2 = 5X_3$, $x_3 = X_1$, (a) calculate the principal stretches λ_1, λ_2, and λ_3 (common to both (U and V); (b) calculate the principal directions $\mathbf{N}_1$, $\mathbf{N}_2$, and $\mathbf{N}_3$ of U; (c) calculate the principal directions $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ of V; (d) verify that the principal directions of U and V differ by a rotation,i.e., $\mathbf{n}_1 = \mathbf{R}\mathbf{N}_1$, $\mathbf{n}_2 = \mathbf{R}\mathbf{N}_2$, $\mathbf{n}_3 = \mathbf{R}\mathbf{N}_3$.

Q.4 In direct notation prove that $\dot{\mathbf{F}} = \mathbf{LF}$.

Section B

Q.5 Starting with the spatial statement of the balance of linear momentum in integral form,

$$\frac{d}{dt}\int_{\mathcal{P}} \rho \mathbf{v} dv = \int_{\mathcal{P}} \rho \mathbf{b} dv + \int_{\partial\mathcal{P}} \mathbf{t} da,$$

derive the corresponding pointwise form

$$\rho \dot{\mathbf{v}} = \rho \mathbf{b} + \mathrm{div}\mathbf{T}.$$

Q.6 What is invariance under superposed rigid body motions (SRBM)? Prove that $\mathbf{t}^+ = \mathbf{Qt}$.

Q.7 Prove in direct notation that

$$\frac{\partial \psi}{\partial \mathbf{F}} \cdot \dot{\mathbf{F}} = \frac{\partial \psi}{\partial \mathbf{F}} \mathbf{F}^{\mathrm{T}} \cdot \mathbf{L}.$$

Q.8 Demonstrate that the Newtonian (fluid) constitutive equation can be combined with conservation of linear momentumm to obtain the Navier-Stokes equations.

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