Exam.Code:1014
Sub. Code: 7749

1128
M. E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Part.

x-x-x

PART A

1. (a) Prove that (5)

$$\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{\sin x}{x} - \cos x$$

(b) Find a basis of solutions by Frobenius method: (5)

$$x^2y'' + 4xy' + (x^2+2)y = 0$$

2. Find a solution of the given differential equation by reduction to the Legendre equation: (10)

$$(a^2 - x^2)y'' - 2xy' + n(n+1)y = 0, \quad a \neq 0$$

3. Find a general solution of the differential equation in terms of J_v and Y_v using the given substitution: (10)

$$y'' + 4x^2y = 0, \quad (y = u\sqrt{x},\ x^2 = z)$$

4. (a) Find the eigen values and eigen functions of the following ordinary differential equation: (5)

$$y'' + \lambda y = 0,\ y(0) = 0,\ y(1) + y'(1) = 0$$

(b) Solve the STRUM-LIOUVILLE problem: (5)

$$y'' + \lambda y = 0, \quad y(0) + y'(0) = 0;\ \ y(\pi) - y'(\pi) = 0$$

PART B

5. Use Runge-Kutta method to find $y(0.2)$ for the equation (10)

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y$$

given that $y = 1$, $\dfrac{dy}{dx} = 0$ when $x = 0$.

6. Using Picard's method, obtain the solution of (10)

$$\frac{dy}{dx} = x(1 + x^3y), \quad y(0) = 3$$

Tabulate the values of $y(0.1),\ y(0.2), \ldots\ldots, y(1.0)$.

7. Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square with $0 \leq x \leq 4$, $0 \leq y \leq 4$ with $u = 0$ on the boundary and mess length $=1$. (10)

8. Solve $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}$; $0 \leq x \leq 1$, $t \geq 0$ under the condition that $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = 2x$ for $0 \leq x \leq 0.5$ and $u(x,0) = (1-x)$ for $0.5 \leq x \leq 1$. (10)

x-x-x