1128

B.E. (Biotechnology) First Semester
MATHS-101: Calculus
(Common to all Streams)

Time allowed: 3 Hours   Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.*

x-x-x

1. (a) Find the limit of the sequence: $a_n = \dfrac{n!}{n^n}$.

   (b) Find the limit if it exists: $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x}{x^2 + y^2 + x}$.

   (c) Define domain, range and level curves of the function $f(x,y) = 1 - |x| - |y|$. Draw its graph also.

   (d) Find the direction derivetive of $f(x,y) = xy + \cos(y)$ at the point $(3,0)$ in the direction of $A = 3i - 4j$.

   (e) If $|a|$ is much greater than $|b|, |c|, |d|$, to which of a, b, c, d is the value of the function $f(a,b,c,d) = ad - bc$ is most sensitive? Justify your answer. $(5 \times 2 = 10)$

PART A

2. (a) Check the convergence of the following series: (3+3)

   (i) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n!}{2^n}$

   (ii) $\sum\limits_{n=2}^{\infty} \dfrac{n}{(\ln n)^{(n/2)}}$

   (b) Check the convergence of the series: (4)

   $$\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2 + 1)}$$

3. (a) Find $\dfrac{\partial w}{\partial x}$ if $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$.   (4)

   (b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the line $y = 2$ and $x = 0$ about the line $x = 4$.   (3)

   (c) Find the length of the curve $x = \dfrac{y^{3/2}}{3} - \sqrt{y}$ from $y = 1$ to $y = 9$.   (3)

4. (a) Show that $w = f(u,v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$, and if $u = (x^2 - y^2)/2$ and $v = xy$, then w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$. (5)

   (b) The temperature at a point (x,y) on a metal plate is $T(x,y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at origin. What are the highest and lowest temperatures encoutered by the ant?   (5)

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PART B

5. (a) Find the volume of the region in the first octant bounded by coordinates planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$. (6)

   (b) Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. (4)

6. (a) Find the outward flux of the field $\mathbf{F} = xz\,\mathbf{i} + yz\mathbf{j} + \mathbf{k}$ across the boundary of the region D: The entire surface of the upper cap cut from the solid sphere $x^2 + y^2 + z^2 \le 25$ by the plane $z = 3$. (5)

   (b) Show that the differential form in the integral is exact and hence evaluate the integral: (5)

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\,dx + z^2/y\,dy + 2z\,\ln y\,dz$$

7. (a) Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$ curvature and torsion for the space curve: (5)

$$\mathbf{r}(t) = \cosh t\,\mathbf{i} - \sinh t\,\mathbf{j} + t\,\mathbf{k}$$

   (b) A particle moves along the top of the parabola $y^2 = 2x$ from left to right at a constant speed of 5 units per second. Find the velocity of the particle as it moves through the point (2,2). (3)

   (c) Find the equation of the tangent plane to the surface $z = \sqrt{y - x}$ at the point (1,2,1). (2)

x-x-x